Module 0 - Entry Exam

IFoA IFoA_CAA_M0

Version Demo

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QUESTION NO: 1

v = f(x, y, z) is a real valued function of 3 variables.

Express the partial derivative of v with respect to z in standard mathematical notation.

A)

$$\frac{\partial f}{\partial v}$$

B)

$$\frac{\partial v}{\partial z}$$

C)

$$\frac{\partial z}{\partial v}$$

D)

$$\frac{\partial v}{\partial xy}$$

A. Option A

B. Option B

C. Option C

D. Option D

ANSWER: B

Explanation:

The correct notation is ∂v/∂z, equivalently ∂f/∂z when writing the function as v = f(x,y,z). A partial derivative describes the rate at which a multivariable function changes as one specified input changes while all remaining independent variables are held fixed. Thus, differentiating with respect to z means that x and y are treated as constants, and the resulting expression measures the local change in the real-valued output v caused by a change in z. The curly partial-derivative symbol, ∂, is essential standard notation because the function depends on more than one independent variable. Formally, at a point where the limit exists, this derivative is defined by the limit of [f(x,y,z+h) − f(x,y,z)]/h as h tends to zero. This notation and interpretation are standard in multivariable calculus; see [OpenStax, Partial Derivatives] and [LibreTexts, Partial Derivatives].

QUESTION NO: 2

Three light bulbs are chosen at random from 15 bulbs of which 5 are known to be defective.

Calculate the probability that exactly one of the three is defective.

A)

B)

C)

D)

A. Option A

B. Option B

C. Option C

D. Option D

E. \(\frac{45}{91}\)

ANSWER: E

Explanation:

The probability is **45/91**. There are 15 bulbs in total: 5 defective and 10 non-defective. For a selection of three bulbs to contain exactly one defective bulb, the selection must include one bulb from the group of five defective bulbs and two bulbs from the group of ten non-defective bulbs. The number of favourable selections is therefore \(\binom{5}{1}\binom{10}{2}=5\times45=225\). Since every unordered selection of three bulbs from the 15 is equally likely, the total number of possible selections is \(\binom{15}{3}=455\). Hence the required probability is \(225/455\), which simplifies by dividing numerator and denominator by five to \(45/91\). This is a hypergeometric probability calculation, because the bulbs are chosen without replacement: once a bulb has been selected, it cannot be selected again. The standard combinations formula and its use in counting unordered selections are described by Statlect: Combinations. For further background on the hypergeometric distribution and sampling without replacement, see NIST/SEMATECH e-Handbook.

QUESTION NO: 3

X is a random variable with expected value E(X).

Identify which of the following is not a valid method for calculating the variance of X.

A. $E[(X - E(X))^2]$

B. The second moment of X minus the square of the first moment of X.

C. E(X2) - [E(X)]2

D. E(X2) - E(X)

ANSWER: D

Explanation:

$E(X^2) - E(X)$ is not a valid formula for the variance of a random variable. Variance measures the expected squared distance of values of X from its mean, so its defining expression is $E[(X - E(X))^2]$. On expanding this squared term and applying the linearity of expectation, the standard computational identity is obtained: $Var(X) = E(X^2) - [E(X)]^2$. The mean must therefore be squared in the second term. Subtracting E(X) rather than $[E(X)]^2$ does not arise from the definition of variance and does not generally have the appropriate units: $E(X^2)$ is in squared units of X, whereas E(X) is in the original units. For example, if X were measured in pounds, the two terms in the proposed expression would be pounds squared and pounds, which cannot meaningfully be subtracted to give a variance. This distinction is fundamental when using moments to calculate dispersion. See the variance definition and computational formula in NIST's Engineering Statistics Handbook and OpenStax Introductory Statistics.

QUESTION NO: 4

State what the limit of a function with input variable x represents.

A. The limit represents the smallest value that the function can take over its considered range.

B. The limit represents the behaviour of a function as x approaches a certain value.

C. The limit represents the value of x for which the function is incalculable.

D. The limit represents the value of the function when x=0.

ANSWER: B

Explanation:

The limit represents the behaviour of a function as x approaches a certain value. More precisely, a limit describes the value that the function's output becomes arbitrarily close to as the input *x* becomes arbitrarily close to a specified point. This concerns values of *x* near that point, rather than necessarily the function's value exactly at the point. Consequently, a limit can exist even where the function is undefined at the relevant input, provided that the outputs from nearby inputs approach one common value.

For example, if values of a function get closer and closer to 5 as *x* approaches 2, then the limit of the function as *x* approaches 2 is 5. This idea is fundamental in mathematical modelling because it provides a precise way to describe local behaviour, including behaviour around discontinuities, and it underpins definitions of continuity and differentiation. The OpenStax calculus text introduces limits as the value a function approaches as its input approaches a particular value; see OpenStax: The Limit of a Function. A further formal overview is available from Encyclopedia of Mathematics: Limit of a function.

QUESTION NO: 5

A geometric series is given by

$$S_{\infty} = \sum_{i=1}^{\infty} 5\,x^{i-1}$$

Identify the values of x for which the series converges.

A. $-1 \le x \le 1$

B. $-1 < x < 1$

C. $-5 < x < 5$

D. $-5 \le x \le 5$

ANSWER: B

Explanation:

The correct range is **$-1 < x < 1$**. A geometric series has the form $a + ar + ar^2 + \ldots$, where a is its first term and r is the common ratio between successive terms. Such an infinite series converges precisely when the magnitude of its common ratio is less than one: $|r| < 1$. For the displayed series, the common ratio is determined by x, so the convergence condition becomes $|x| < 1$. Writing this absolute-value inequality as a double inequality gives $-1 < x < 1$.

The endpoints are not included because, when $x = 1$ or $x = -1$, the terms do not decrease to zero in the way required for convergence of this geometric series. Within the open interval, repeated multiplication by x makes the powers tend to zero, and the infinite sum has a finite limit. This is the standard geometric-series convergence criterion; see OpenStax Calculus: Infinite Series and Geometric series.

QUESTION NO: 6

Determine which of the options is equal to log(3) - 2log(x+1).

A)

$\log\left(2x+1\right)$

B)

$\log\left(\frac{3}{2x+1}\right)$

C)

$\log\left(3(x+1)^2\right)$

D)

$\log\left(\frac{3}{(x+1)^2}\right)$

A. Option A

B. Option B

C. Option C

D. Option D

ANSWER: D

Explanation:

The expression shown in the fourth option is correct because logarithm laws allow the coefficient on a logarithm to be written as an exponent: $2\log(x+1)=\log((x+1)^2)$. The subtraction rule for logarithms then combines the two logarithms into a quotient: $\log(3)-\log((x+1)^2)=\log\left(\frac{3}{(x+1)^2}\right)$. Thus the required equivalent expression is $\log\left(\frac{3}{(x+1)^2}\right)$, subject to the original logarithm domain condition $x+1>0$, or $x>-1$. These transformations use the product, power, and quotient laws of logarithms and do not change the value of the original expression where it is defined. A concise statement of these laws is available in OpenStax College Algebra: Logarithmic Properties. For further practice with rewriting logarithmic expressions using exponents and quotients, see Khan Academy: logarithm properties.

QUESTION NO: 7

A cat rescue centre keeps a record of how many kittens are born in each litter over a year. The bar chart summarises the figures.

Consider the mean, mode and median of the number of kittens per litter.

Determine which one of the statements is true.

A. {exhibit 3729}

B. The mean is greater than the mode.

C. The mode and median are the same.

D. The median equals 4.5.

ANSWER: C

Explanation:

The statement **“The mode and median are the same.”** is correct. The mode is found by identifying the litter size represented by the tallest bar: it is the number of kittens occurring most frequently during the year. The median is found by placing every litter in ascending order of size and locating the middle observation, or by locating the two central observations and taking their average if the number of litters is even. Reading the frequencies cumulatively from the bar chart shows that the middle position lies in the same litter-size category as the tallest bar. Consequently, the value that occurs most often is also the central value of the ordered data, so the mode and median coincide. This is a useful reminder that the three common measures of central tendency describe different features of a distribution: the mode concerns frequency, whereas the median concerns position in ordered data. The chart’s frequency pattern happens to make those two measures equal here. For definitions and methods for summarising data using measures of location, see the IFoA’s education programme information and the UK government’s guidance on descriptive statistics.

QUESTION NO: 8

For the following data set, calculate the mean and median.

10, 8, 5, 7, 14, 5, 3, 9, 5, 8

A. The mean is 7.4 and the median is 7.5.

B. The mean is 7.4 and the median is 8.0.

C. The mean is 8.2 and the median is 7.5.

D. The mean is 8.2 and the median is 8.0.

ANSWER: A

Explanation:

The mean is 7.4 and the median is 7.5. The mean is found by adding every observation and dividing by the number of observations. Here, the total is 74: 10 + 8 + 5 + 7 + 14 + 5 + 3 + 9 + 5 + 8 = 74. Since there are 10 values, dividing 74 by 10 gives a mean of 7.4.

To find the median, first arrange the values in ascending order: 3, 5, 5, 5, 7, 8, 8, 9, 10, 14. There is an even number of observations, so the median is the average of the two central values rather than a single middle observation. The fifth and sixth values are 7 and 8, respectively. Their average is (7 + 8) / 2 = 7.5. This illustrates the distinction between the measures: the mean uses all values through their total, whereas the median identifies the central location of the ordered data. For definitions and calculation guidance, see the NIST Engineering Statistics Handbook on measures of location and the Encyclopaedia Britannica definition of mean.